ECE 321 – PRINCIPLES OF COMMUNICATION

PHASE MODULATION (PM) STUDY GUIDE

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I. OBJECTIVES

1. Define Phase Modulation and explain its core concept in your own words.
2. Derive and understand the mathematical expression for a PM wave.
3. Differentiate between Phase Modulation (PM) and Frequency Modulation (FM) in terms of definition, mathematical relationship, and practical implications.
4. Analyse the spectrum of a PM wave and understand the concept of the Modulation Index (β) and Carson's Bandwidth Rule.
5. Draw block diagrams for PM generation (Direct and Indirect Methods) and demodulation.
6. List the key advantages and disadvantages of PM compared to other modulation schemes.

2. PREREQUISITE KNOWLEDGE

Before diving in, ensure you are comfortable with:

a) **Basic Sinusoids:** $A\cos(2\pi ft + \phi)$

b) **Amplitude Modulation (AM):** Concepts of carrier wave, modulating signal, and sidebands.

c) **The Concept of "Angle":** Understanding that the argument of the cosine function, $\theta(t) = 2\pi ft + \phi$, is the "angle" or "instantaneous phase."

d) **Differentiation and Integration:** Basic calculus is essential for the PM-FM relationship.

3. CORE CONCEPTS

3.1. What is Phase Modulation?

- **Definition:** Phase Modulation is a technique where the **phase angle (ϕ)** of the carrier signal is varied linearly with the message signal m(t).
- **Key Idea:** The information is contained in the **phase shifts** of the carrier wave.

3.2. The Mathematical Representation

1. **Carrier Wave:** $c(t) = Ac \cos(\theta_i(t)) = A_c\cos(2\pi f_c\, t + \phi_0)$

- A_c: Carrier Amplitude (constant)
- $\theta_i(t)$: Instantaneous Phase Angle
- f_c: Carrier Frequency
- ϕ_0: Initial Phase (often assumed to be 0)

2. **Phase Modulated Wave:**

   In PM, the instantaneous phase is directly proportional to the message signal:

   $$\theta_i(t) = 2\pi f_c\, t + k_p\, m(t)$$

   - k_p is the **Phase Sensitivity** (or Phase Deviation Constant) of the modulator, in radians per volt (rad/V).

3. **Final PM Wave Equation:**

   $$s(t) = A_c \cos(\, 2\pi f_c\, t + k_p\, m(t)\,)$$

   - The term $k_p\, m(t)$ is the **instantaneous phase deviation** from the carrier's phase.

C. THE CRUCIAL DISTINCTION: PM VS. FM

This is the most important and often tested concept. Don't just memorize the difference; understand the relationship.

FEATURE	PHASE MODULATION (PM)	FREQUENCY MODULATION (FM)
Definition	Instantaneous Phase is proportional to m(t).	Instantaneous Frequency is proportional to m(t).
Mathematical Form	$s_{PM}(t)$ = Ac cos(2πfc t + **k_p m(t)**)	$s_{FM}(t)$ = Ac cos(2πfc t + **$2\pi k_f \int m(\tau)d\tau$**)
What Varies?	Varies the phase of the carrier.	Varies the frequency of the carrier.
Relationship	A PM wave can be generated by first integrating the message signal and then using it to phase modulate a carrier.	An FM wave can be generated by first differentiating the message signal and then using it for phase modulation.
Key Insight	**The derivative of the phase of a PM wave is proportional to the derivative of m(t).**	**The derivative of the phase of an FM wave is proportional to m(t) itself.**

Memory Aid: "You get **FM** from **PM** by putting an **I**ntegrator before the modulator." (FMI). Conversely, "You get **PM** from **FM** by putting a **D**ifferentiator before the modulator."

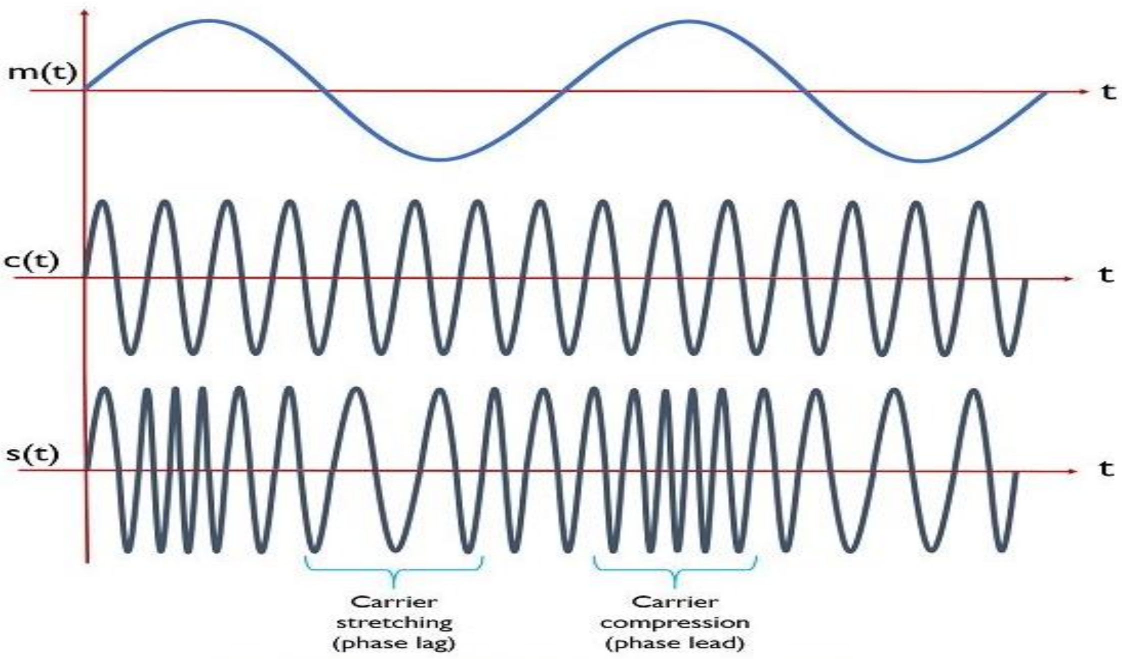

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Figure 1. Single-tome Modulated PM wave

4. IN-DEPTH ANALYSIS

4.1. Modulation Index for PM (β)

- The modulation index describes the **maximum deviation** of the phase.
- For a single-tone modulating signal: $m(t) = A_m \cos(2\pi f_m t)$

$$\beta = k_p * A_m \text{ (unitless, in radians)}$$

- **Interpretation:** β_p represents the **peak phase deviation** in radians. A β of 1 radian means the carrier's phase shifts by a maximum of 1 radian from its unmodulated value.

4.2. Bandwidth of a PM Wave

Like FM, PM produces an infinite number of sidebands, but their significant power is contained within a finite bandwidth.

4.2.1 Carson's Bandwidth Rule (an excellent approximation):

$$BW \approx 2(\beta + 1)f_m \text{ for a single-tone message.}$$

Where f_m is the maximum frequency component of the message signal. For a complex m(t), replace f_m with the message bandwidth B.

$$BW \approx 2(\Delta\phi_p / (2\pi) + 1)B = 2((k_p \max|m(t)|) / (2\pi) + 1)B$$

4.2.2 Key Point:
The bandwidth depends on both the amplitude of the message *max|m(t)|* and its frequency (f_m or B). Higher β (deeper modulation) leads to a wider bandwidth.

4.3. Spectrum

- The spectrum of a single-tone PM wave is identical in form to that of an FM wave. It consists of a carrier component at f_c and an infinite number of sidebands at $f_c \pm nf_m$.
- The amplitudes of these spectral components are given by **Bessel functions of the first kind** J_n(β).
- The power of the PM wave remains constant and is equal to the power of the unmodulated carrier ($A_c^2/2$), regardless of the modulation index. Power is just redistributed between the carrier and the sidebands.

5. IMPLEMENTATION

5.1. Generating a PM Signal

5.1.1 Direct Method

Uses a voltage-controlled phase shifter (e.g., a Varactor Diode in a tuned LC oscillator circuit). The message voltage m(t) directly controls the phase shift. This method is less stable for maintaining a precise centre frequency.

5.1.2 Indirect (Armstrong) Method

The more common and stable approach.

Step 1: Generate a stable, low-power NBFM signal (with a very low modulation index).

Step 2: Use frequency multipliers to increase both the carrier frequency f_c and the modulation index β to the desired values.

(Tip: Practice drawing this block diagram)

5.2. Demodulating a PM Signal

- The goal is to extract m(t) from the phase variations of s (t).
- The most common method is using a **Phase-Locked Loop (PLL)**.
    - The PLL's Voltage-Controlled Oscillator (VCO) locks onto the frequency and phase of the incoming PM signal.
    - The control voltage needed to keep the VCO locked is proportional to the original message signal m(t).
- A **Frequency Discriminator** (followed by an integrator) can also be used, as the output of a discriminator is proportional to the instantaneous frequency, which for PM is d/dt [k_p m(t)]. Integrating this output recovers m(t).

6. APPLICATIONS, ADVANTAGES, AND DISADVANTAGES

6.1 Applications

- **Wireless Communications:** Used in various digital modulation schemes (BPSK, QPSK, etc.), which are fundamentally forms of digital phase modulation.
- **Signal Generation:** In instrumentation and synthesizers.
- **Data Modems.**

6.2 Advantages (over AM):

- **Constant Envelope:** Makes it resistant to amplitude noise and fading.
- **Power Efficient:** Transmitters can use efficient non-linear amplifiers (Class C).
- **Improved SNR:** Exhibits a capture effect, where a stronger signal will "capture" the receiver.

6.3 Disadvantages

- **Requires more complex circuitry** than AM receivers.
- **Requires more bandwidth** than AM (a trade-off for the improved performance).

7. PRACTICE AND SELF-ASSESSMENT

7.1 Conceptual Questions

1. If the amplitude of the message signal m(t) is doubled, what happens to the modulation index β_p and the bandwidth of the PM wave?
2. Explain why PM is considered a "non-linear" modulation technique.
3. How would you convert a PM modulator into an FM modulator? Draw a block diagram.
4. Why is the bandwidth of a PM signal dependent on both the amplitude and the frequency of the message signal?

7.2 Calculation Problems

1. A carrier $100\cos(2\pi 10^8 t)$ is phase modulated by the signal $10\cos(2\pi 10^3 t)$. If the phase sensitivity k_p is 1 rad/V, find:
   a) The modulation index β.
   b) The expression for the PM wave s(t).
   c) The approximate bandwidth using Carson's rule.
2. A message signal has a bandwidth of 5 kHz and a peak amplitude of 2 V is used to phase modulate a carrier with k_p = 1.5 rad/V. Estimate the required transmission bandwidth.

7.3 Practice Drawing

1. Draw the block diagram for the Armstrong (Indirect) method of generating a PM wave.
2. Draw the block diagram for a PLL-based PM demodulator.